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Ionization of hydrogen atom by X-ray absorption in the presence of optical laser field

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Abstract : The absorption of X-rays in hydrogen atom considering the irradiation of the target by an intense optical laser of frequency ω is studied. We find that the terms of the modified scattering amplitude has different dependence on polarization vectors of X-ray field and laser fields. There is resonance in the differential cross section for absorption at different frequencies when ω (the laser frequency) becomes nearly equal to atomic transition frequency.

Keywords : Ionization, hydrogen atom, X-ray absorption, optical laser field.

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1. Introduction

With the availability of intense laser sources, recently there has been great increase of interest in the studies of multiphoton processes in atoms (Kaiser and Garret 1961, Reiss 1971, Freund 1973, Mittleman 1974, Gersten and Mittleman 1974, Ehlotzky 1975, Sarkar and Chakraborty 1988), plasmas (Silin 1965, Seely and Harris 1973, Shima and Yaton 1975) and solids (Jones and Reiss 1977, Nunes 1983, Tronconi and Nunes 1985a, b). In this work, we are studying the modification of the differential cross section for absorption of X-rays in hydrogen atom due to the simultaneous irradiation of the target by an intense optical laser of frequency ω which is taken to be plane polarized. The absorption of X-rays in an atom placed in an intense optical laser field is of interest in the study of laser produced plasmas, where X-rays are produced as a result of the interaction of the laser beam with matter (Jain and Tzoar 1977). Considering the electromagnetic interaction between the X-ray photon and the electron (described by $-\frac{e}{mc}A^{x}.p$ term or its equivalent form of the dipole type interaction $r.\epsilon$ generated by the gauge transformation $\exp[-ie\mathcal{A}(t).r/\hbar c]$) to be perturbative, we have evaluated transition probabilities using laser-modified initial bound state wave function and laser-dressed final continuum wave function. Such wave function has been used by Joachain *et al* (1988) in a corresponding investigation when X-ray in the above problem is replaced by a fast projectile electron which is scattered by the target due to Coulomb interaction. We may also think that the projectile electron emits a virtual photon which then interacts with the target atom. We may note that Fonseca and Nunes (1988) in their recent study of the above problem have taken the initial bound 1s-state to be unperturbed in the laser field while the final wave function of the ejected electron to be Volkov type laser dressed plane wave.

2. Theory

The Hamiltonian of the target atom in the presence of the laser field can be written as

$$H=\frac{1}{2m}\left[p+\frac{e}{c}A(t)\right]^2+U(r) \tag{1}$$

where the vector potential $A(t)$ is given by

$$A(t)=A^L(t)+A^X(t)$$

and $U(r)$ is the potential.

The superscript 'L' and 'X' refer lo the laser field and X-ray field.

Considering the following gauge transformation

$$\Psi'(r, t)=\exp\left[-\frac{ieA(t)}{\hbar c}.r\right]\Psi(r, t), \tag{2}$$

we have

$$i\frac{\delta\Psi(r, t)}{\delta t}=(H^{(0)}+H^{(1)})\Psi(r, t) \tag{3}$$

where

$$H^{(0)}=\frac{1}{2m}p^2+U(r). \tag{3a}$$

The perturbation Hamiltonian $H^{(1)}$ is given by

$$H^{(1)}=r.\epsilon(t)=r.\epsilon^X(t)+r.\epsilon^L \tag{4}$$

where ϵ is the electric field. As the wavelengths of both laser field and X-ray are taken to be very large compared to the dimension of the atom we ignore the space dependence of the vector potentials A^X and A^L. This assumption leads to the dipole type interaction given above. Henceforth we omit the superscript 'L' of A^L. The occurrence of the two terms in the above dipole type interaction Hamiltonian $H^{(1)}$ implies that we can study the present problem in the manner of two potential problem formulated by Golderger and Watson (1964).

The transition probability for ionization is given by

$$W=\int\frac{d^3p}{(2\pi\hbar)^3}\lim_{T\to\infty}\left\{|S_{fi}|^2/\int_{-T}^{T}dt\right\} \tag{5}$$

where the S-matrix element can be written as

$$S_{if}=(ih)^{-1}\int_{-T}^{T}dt\left[\langle\Psi^{(-)}_{f(X,L)}\mid \boldsymbol{r}.\epsilon^{X}\mid\Psi^{+}_{i(L)}\rangle+\langle\Phi_f\mid\boldsymbol{r}.\epsilon^{L}\mid\Psi^{+}_{i(L)}\rangle\right]. \tag{6}$$

In the above relation, the wave function $\Psi_{f(X,L)}$ corresponds to the case when the effects of both X-ray field and laser field are considered for electron wave function in the final state. In the construction of $\Psi_{i(L)}$ (laser dressed initial state wave function) we only consider the effect of laser field. Φ_f corresponds to bare field wave function. As the intensity of the laser field is taken to be very large compared to that of X-ray field we can replace $\Psi_{f(X,L)}$ by $\Psi_{f(L)}$ and then omit the subscript '*L*' (signifying laser modified wave function). The second term in the expression for S_{if} cannot induce the transition (related to ionization of electron by X-ray) considered in our problem and is therefore ignored. This is due to the fact that second term does not involve X-ray field.

The electric field associated with the laser field which in general can be taken to be plane polarized is given by

$$\epsilon=-\frac{1}{c}\frac{\partial A}{\partial t}=\epsilon_0\sin\omega t. \tag{7}$$

Then the dipole-type interaction can be written as

$$H^{(1)}=\boldsymbol{r}.\epsilon(t) \tag{8}$$

$$=M^{1,1}e^{i\omega t}+M^{1,-1}e^{-i\omega t} \tag{8a}$$

where

$$M^{1,\pm1}=\mp\frac{i}{2}\boldsymbol{r}.\epsilon_0.$$

In the high energy limit, we approximate the laser modified continuum Coulomb wave function $\Phi_{k_B}(\boldsymbol{r},t)$ of the ionized electron of momentum $\boldsymbol{k}_B$ by the corresponding laser modified plane wave function of momentum $\boldsymbol{k}_B$. In the high energy region where energy of X-ray photon is very much large compared to the binding energy of atomic electron, effect of Coulomb distortion becomes quite small and we have

$$\Psi^{(-)}_{f(X,L)}\sim\Phi_{k_B}(r,t)=\frac{1}{(2\pi)^{3/2}}\exp(-iE_{k_B}t)\exp(i\boldsymbol{k}_B\cdot\boldsymbol{r})$$

$$\exp(-i\boldsymbol{k}_B\cdot\boldsymbol{\alpha}_0\sin\omega t)\times\exp\left(-\frac{i}{h}\frac{e^2}{2mc^2}\int^{t}A^2(\tau)d\tau\right) \tag{9}$$

The lower limit of integration in eq. (9) is $t=-\infty$ but in actual experimental situation $A(\tau)$ vanishes much before $t=-\infty$. In our approximate result for cross section this integral has no effect.

The initial dressed wave function $\Psi^{+}_{i(L)}=\Phi_0(r,t)$ is given by

$$\Phi_0(r,t)=\exp(-iE_0t)\exp(iA.r)\Big[\Psi_0(r)+i\sum_n\Big\{\frac{\exp(i\omega t)M^{1,1}_{n0}}{E_n-E_0+\omega}-\frac{\exp(-i\omega t)M^{1,1}_{n0}}{E_n-E_0-\omega}\Big\}\times\Psi_n(r)\Big] \quad (10)$$

where

$$M^{1,\pm1}_{n0}=\langle\Psi_n\mid M^{1,\pm1}\mid\Psi_0\rangle$$

In eq. (10) we only consider the effect of first order perturbation terms having resonance like structure. Since we are mainly interested in the cross section near resonance we replace $\exp(-iA.r)=1-iA.r\cdots$term in eq. (10) by 1 in the evaluation of S-matrix element. The form of dipole type interaction (occurring in eq. (8)) has been derived by Delone and Krainov (1984) using the gauge transformation defined by eq. (2) and assuming that the variation of the vector field A in the range of atomic dimension is negligible. We may note that in the usual interaction term $-\frac{1}{c}A.j$ (j is the current density) if we retain the first term of plane wave expansion of A, one obtains electric dipole contribution as discussed by Rose (1967). In the above expression, Ψ_n is the target state of energy E_n and Ψ_0 is the target ground state of energy E_0.

In the new notation S-matrix element is

$$S=-i\int_{-\infty}^{\infty}dt\,\langle\Phi_{k_B}(r,t)\mid r.\epsilon^{I}(t)\mid\Phi_0(r,t)\rangle. \quad (11)$$

If l is the number of laser-photons transferred, we can write

$$S=(2\pi)^{-1}i\sum_{l}^{\infty}d(E_{k_B}-E^{I}-E_0-l\omega)f^{l} \quad (11a)$$

where f^{l} is the first Born approximation to the reaction amplitude with the transfer of l photons and is given by

$$f^{l}=(f^{l}_{I}+f^{l}_{II}) \quad (12)$$

$$f^{l}_{I}=J_l(\lambda)\langle k_B\mid\epsilon^{I}.r\mid\Psi_0\rangle \quad (12a)$$

$$f^{l}_{II}=-i\sum_{n,l\,m}\langle k_B\mid\epsilon^{I}.r\mid\Psi_{nlm}\rangle\times\Big[\frac{J_{l-1}(\lambda)M^{1,1}_{nlm,0}}{E_n-E_0-\omega}-\frac{J_{l+1}(\lambda)M^{1,-1}_{nlm,0}}{E_n-E_0+\omega}\Big]. \quad (12b)$$

In the above expressions

$$| k_B \rangle = \frac{1}{(2\pi)^{3/2}} e^{i k \cdot r}$$

$$\lambda = -(k_B . \epsilon^L)\omega^{-1}.$$

We have

$$f_I^l = J_l(\lambda)(2\pi)^{-3/2} \frac{N_{10}}{2\sqrt{\pi}} \left(\epsilon^X . i \frac{\partial}{\partial k_B}\right) \times \left(-\frac{\partial}{\partial \beta}\right) \alpha \Big|_{\beta=1} \tag{13}$$

where

$$\alpha = \frac{4\pi}{\beta^2 + k_B^2}$$

$$= iJ_l(\lambda) \frac{(2\pi)^{-3/2}}{2\sqrt{\pi}} 4\pi N_{10} \left(-\frac{\partial}{\partial \beta}\right) \left(-\frac{2(\epsilon^X . k_B)}{(\beta^2 + k_B^2)^2}\right) \tag{13a}$$

and

$$f_{II}^l = -i(2\pi)^{-3/2} N_{21} I_{21} 4\pi \left[\frac{J_{l-1}(\lambda)}{E_2 - E_0 - \omega} - \frac{J_{l+1}(\lambda)}{E_2 - E_0 + \omega}\right] R'_{21} \left(-\frac{\partial}{\partial \beta}\right)$$

$$\left[\frac{2(\epsilon^X, \epsilon^L)}{(\beta^2 + k_B^2)^2} - \frac{8(k_B, \epsilon^X)(k_B, \epsilon^L)}{(\beta^2 + k_B^2)^3}\right] \Big|_{\beta=\frac{1}{2}} - i(2\pi)^{-3/2} N_{31} I_{31}$$

$$\times 4\pi \left[\frac{J_{l-1}(\lambda)}{E_3 - E_0 - \omega} - \frac{J_{l+1}(\lambda)}{E_3 - E_0 + \omega}\right] R'_{31} \left(-\frac{\partial}{\partial \beta}\right) \left[\frac{2(\epsilon^X, \epsilon^L)}{(\beta^2 + k_B^2)^2}\right.$$

$$\left. - \frac{8(k_B, \epsilon^X)(k_B, \epsilon^L)}{(\beta^2 + k_B^2)^3}\right] \Big|_{\beta=\frac{1}{3}} + \cdots \tag{14}$$

The first Born approximation result is quite justified in the high energy limit. Further, this provides us with a closed analytic expression showing clearly the dependence of the cross section on energy, scattering angles and photon polarizations.

We write hydrogenic wave function in the form

$$\Psi_{nlm} = N_{nl} e^{-r/n} R'_{nl}(r) Y_{lm} = R_{nl}(r) Y_{lm}. \tag{15}$$

In the above expression

$$I_{nl} = \int \Psi^*_{nlm} \Psi_0 Y_{lm} dr \tag{15a}$$

which does not depend upon azimuthal quantum number m.

3. Results and discussion

The terms f_I^l and f_{II}^l of the scattering amplitude has different dependence on polarization vectors of X-ray field and laser fields described below.

(1) When ϵ^X is perpendicular to k_B i.e. $(\epsilon^X \perp k_B)$, f_I^l term given by eq. (13) always vanishes unlike f_{II}^l term.

(2) When ϵ^L of laser field is perpendicular to k_B i.e. $(\epsilon^L \perp k_B)$ the term f_I^l vanishes for $l \neq 0$ but the term f_{II}^l is still non-vanishing for $l=1$ due to the presence of the term (ϵ^X, ϵ^L) in the expression (14) for f_{II}^l.

It appears that when the energy of laser-photon (ω) is nearly equal to the energy difference between 2s state and ground state $(E_2 - E_0)$ or that between 3s state and ground state $(E_3 - E_0)$ there is a sharp rise in the differential cross section. This

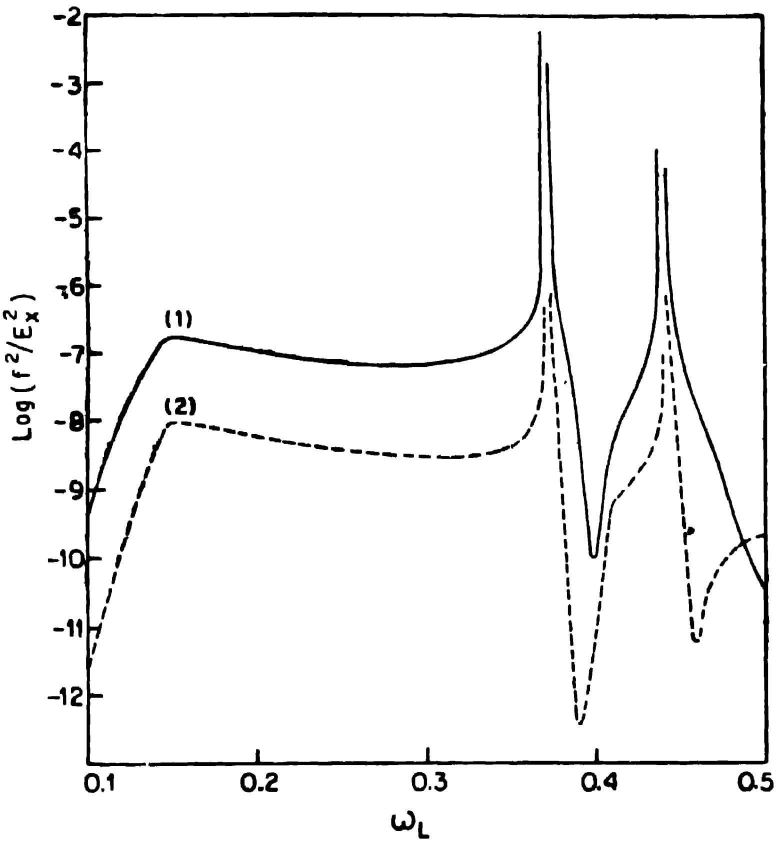

Figure 1. Log of differential cross section divided by $E_X^2 = (\epsilon^X)^2$ is plotted as a function of laser frequency ω_L when the direction of ionized electron momentum k_B is parallel to the direction of polarization vectors characterizing both X-ray and laser field (field strength of 0.02 a.u). Curves 1 and 2 correspond to X-ray energy of 10 a.u and 20 a.u respectively.

also holds in general whenever ω equal $E_n - E_0$ for any values of n (principal quantum number). In the above case f_{II}^l term obviously dominates over f_I^l

term. We may note that f^{l}_{II} term arises due to the laser modification of the ground state of the hydrogen atom.

For other values of ω and arbitrary directions of $k_B, \epsilon^X, \epsilon$, the term f^L_I involving $J_l(\lambda)$ factor can be made almost to vanish by choosing appropriate field strength of the laser field. In this case also the f^l_{II} term dominates over the f^l_I term. In principle, appropriate combination of laser frequency ω and field strength can be taken so that $J_l(\lambda)$ factor of f^l_I has nodes for certain values of the argument λ.

In Figure 1 we have plotted log of differential cross section divided by $E^2_X = (\epsilon^X)^2$ as a function of laser frequency ω_L when the direction of ionized electron momentum k^B is parallel to the direction of polarization vectors charac-

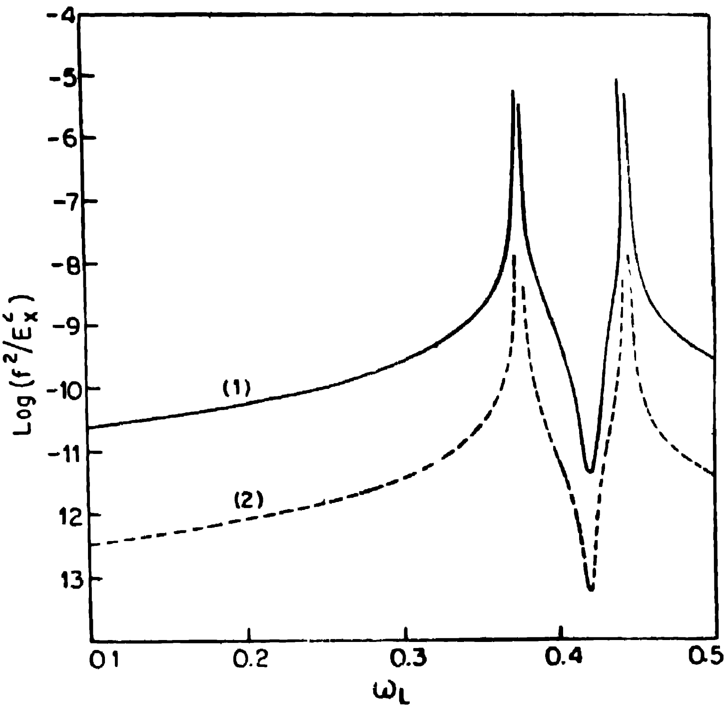

Figure 2. The same as in Figure 1 with the direction of ionized electron momentum k_B is perpendicular to the direction of polarization vectors characterizing both X-ray and laser fields.

terizing both X-ray and laser fields for two different X-ray energies of 10 and 20 a.u. Figure 2 represents the same when momentum k_B is perpendicular to the direction of polarization vectors characterizing both X-ray and laser fields. Comparing Figures 1 and 2 we find that there is a weak broad maximum in the differential cross section when k_B is parallel to the direction of polarization vectors unlike in case when k_B is perpendicular to the direction of polarization vectors.

From the numerical results represents by the graphs of Figures 1 and 2, we find resonance like structure in the differential cross section as discussed before. In tha above numerical calculation, the laser field strength is considered to be 0.02 a.u.

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